Thermodynamics of black holes: an analogy with glasses

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(July 12, 1998; e-mail: nieuwenh@phys.uva.nl)

The present equilibrium formulation of thermodynamics for black holes has several drawbacks, such as assuming the same temperature for black hole and heat bath. Recently the author formulated non-equilibrium thermodynamics for glassy systems. This approach is applied to black holes, with the cosmic background temperature being the bath temperature, and the Hawking temperature the internal temperature. Both Hawking evaporation and absorption of background radiation are taken into account. It is argued that black holes did not form in the very early universe.

04.70-s, 04.70 Dy, 05.70-a, 71.55 Jv, 97.60 Lf

Black holes are singular cosmological objects, with very strong gravitational forces. Nothing, even light, can escape from it at sizeable rates. Many indications point at their presence in the universe. In the center of our own galaxy there is probably a black hole.

Thermodynamics is the old science that describes the energy balance of systems, ranging from steam machines to crystals and stars. It has been a challenge to find out whether an uncommon object as a black hole is governed by these universal laws. Attempts in this direction, started by Bekenstein [1], will be reviewed. For introductory texts on the subject, see e.g. [2] [3] [4]. A solution will be proposed that is based on the newly formulated thermodynamics for glasses.

A black hole has "no hair", i.e. it can be characterized by a few parameters, namely its mass M, charge q and angular momentum J. This is reminiscent of fluids, that can be characterized by temperature and pressure. It is known for long that the energy $U = Mc^2$ satisfies [5]

$$\mathrm{d}U = \frac{\kappa}{8\pi}\mathrm{d}A + \Omega\cdot\mathrm{d}J + \phi\mathrm{d}q \tag{1}$$

where κ is the surface tension, $A = 4\pi R_s^2$ the area expressed in the Schwarzschild radius R_S, Ω the horizon's angular velocity, and ϕ the electrostatic potential at the horizon. This law holds when adding matter to one given black hole, but also when comparing two different black holes. These two very different applications suggest a universal validity, and a thermodynamic description.

Classical black holes cannot decrease their surface area [6], a property reminiscent of the entropy of a closed system. This analogy motivated Bekenstein [1] 25 years ago to formulate the laws of black holes mechanics in a thermodynamic framework. He introduced as entropy the area in dimensionless units, so divided by the square of Planck's length $L_P = \sqrt{\hbar G/c^3}$. There was still freedom to choose a multiplicative constant, now known to be $k_B/4$. This leads to the "information" entropy

$$S_{BH} = \frac{k_B A}{4L_P^2} = \frac{\pi R_S^2 k_B c^3}{\hbar G} \tag{2}$$

The presence of $\hbar$ calls for a quantum mechanical interpretation. Not much later Hawking demonstrated the quantum evaporation of black holes [7]. This underlined the relevance of Bekenstein's approach. The black hole radiates as a black body at Hawking temperature

$$T_H = \frac{\hbar G\kappa}{2\pi c^3 k_B} = \frac{\hbar c^3}{8\pi G M k_B} \tag{3}$$

where the second equality holds for a non-rotating, neutral black hole, having $R_S = 2GM/c^2$. All possible particles are emitted at this temperature; for large black holes, however, T_H is so small, that in practice only massless particles (photons, neutrino's, gravitons) are emitted.

Between these two fundamental steps, Bardeen, Carter and Hawking [5] had formulated "the four laws of black hole dynamics". The zeroth law states that the surface tension κ is constant at the black hole surface, just like the temperature is the same everywhere in an equilibrium system. The first law is given in eq. (1). Since the last two terms corresponds to work terms, one may write this relation in the suggestive form

$$\mathrm{d}U = T_H\mathrm{d}S_{BH} + \text{đ}W \tag{4}$$

This formulation is sometimes referred to as the first law of black holes thermodynamics. Bekenstein had also discussed the *generalized second law*

$$\mathrm{d}S_{BH} + \mathrm{d}S_m \geq 0 \tag{5}$$

where S_m is the entropy of the matter outside the black hole. The third law states that "optimal" black holes, the ones that have $\kappa = T_H = 0$, cannot be reached by a finite number of steps [5] [8].

From the view point of a condensed matter physicist, the literature on black hole thermodynamics is somewhat confusing. First of all, one should define the system for which a thermodynamic description is to be given. This is rarely done; we shall see below what consequences this has. A natural choice is to consider as system the black hole and a "Gedanken" sphere around it of, say, a hundred times the Schwarzschild radius. One could also consider the whole universe as an isolated container.

Our next objection concerns the formulation of the first law in black hole literature. The standard formulation is

$$\mathrm{d}U = \text{đ}Q + \text{đ}W \tag{6}$$

saying that the increase of the system energy equals the heat added to the system and the work done on the system. đQ has to be determined for the system under consideration. The second law only says that heat must flow from high to low temperatures, which requires that

$$\text{đ}Q \leq T\mathrm{d}S \tag{7}$$

The equality sign holds if and only if there is equilibrium. It is seen that eq. (4) *is not the first law of thermodynamics*, it is obtained from it after identifying $T = T_H$, $S = S_{BH}$ and inserting đ$Q = T_H \mathrm{d}S_{BH}$ from the second law. This may seem plausible, but it is not: one has assumed equilibrium of the whole system at temperature T_H, which is not present. It is indeed well known that the "Bekenstein" specific heat $C_{\mathrm{Bek}} \equiv \mathrm{d}U/\mathrm{d}T_H$ is negative, so even if equilibrium at T_H were present, fluctuations would drive the system away from it. As eq. (4) follows from solving dynamical equations, there is nothing wrong with it, but *it has no foundation within standard thermodynamics.*

In black hole literature it is often stated that the entropy cannot decrease. Let us recall that eq. (7) only requires that for a closed system.

Having defined the system, one should discuss its entropy. For the Gedanken sphere with the black hole in it, eq. (6) applies. Notice that the entropy of eq. (7) is the one that belongs to the same system, $S = S_{BH} + S_m^{\mathrm{Gs}}$. The latter is the entropy of the cosmic background matter outside the black hole but inside the Gedanken sphere, and expected to scale with the sphere's volume. There is no justification for including the entropy (or the energy) of matter outside the sphere. The radiation generated by the hole will quickly leave the system and go to the heat bath around it; this is described by a đ$Q < 0$.

If, on the other hand, the whole universe is considered as system, then đ$Q = 0$. If no work is done, this implies that $\mathrm{d}U = 0$, saying that energy radiated from the hole is still inside the system. In that case eq. (4) does not describe the change of the system's energy, it only says something about the black hole. The total entropy is now $S = S_{BH} + S_m$, and the second law indeed says that $\mathrm{d}S \geq 0$.

We conclude that eq. (4) and (5) should not be applied simultaneously: they refer to different cases. In practice this means: different time scales. When only the black hole and its Gedanken sphere are considered, this describes the radiation emitted in a time $\mathrm{d}t$. When considering the change in entropy of the whole universe, one tacitly assumes time scales so large that the emitted radiation has come in equilibrium.

A final, severe, objection against the current formulation of thermodynamics for black holes is: *what is the heat bath?* By considering T_H within thermodynamics, this is by definition the bath temperature, and normally also the temperature of the object. This can only apply to a black hole in equilibrium with its own Hawking radiation, which is an unstable and thus unphysical situation; it can also not deal with black holes of different size. Physically there is one, and only one choice for the bath: for a black hole that has swallowed all matter around it, the heat bath is the cosmic background radiation, that presently has temperature $T_{\mathrm{cb}} = 2.73\,K$. So the actual problem deals with a system of which the dynamics prefers to "live" at a second temperature, namely T_H. This calls for a two-temperature description.

Recently the author has proposed a thermodynamic description of the glassy state [9] [10] [11]. The essential point is that, as there is no equilibrium, time has to be kept as additional parameter. Within thermodynamics a more useful extra variable is the effective temperature $T_e(t)$. Whereas the fast processes are at equilibrium at the bath temperature T, the slow or configurational modes are at a quasi-equilibrium at $T_e(t)$. In glasses $T_e(t)$ exceeds T. Indeed, in the glass formation process by cooling from high temperatures, $T_e(t)$ has lost track of $T(t)$, and is since then lagging behind, trying to reach it in the very remote future. By eliminating t one may specify the cooling trajectory by a function $T_e(T)$. By doing smoothly related cooling experiments at a set of pressures p_i one defines a surface $T_e(T, p)$ in (T, T_e, p)-space. To cover that space, many experiments are needed, e.g. at different pressures and cooling rates. Alternatively, one could keep one given system under fixed external conditions, and consider its aging behavior. These two options are quite analogous to the ones for black holes [5], mentioned below eq. (1).

The fast and slow modes do not only have their own temperature, they also have their own entropy. The fast modes have the *entropy of equilibrium processes* S_{ep}, while the slow modes involve the "configurational" or "information" entropy or "complexity" $\mathcal{I}$. The total entropy is $S = S_{\mathrm{ep}} + \mathcal{I}$. The basic point has been the expression for the change in heat

$$\text{đ}Q = T\mathrm{d}S_{\mathrm{ep}} + T_e \mathrm{d}\mathcal{I} \tag{8}$$

which satisfies (7) since $T_e > T$ and $\mathrm{d}S_{BH} < 0$. The latter holds since in the course of time the system will go to lower, less degenerate modes. In combination with the first law this yields the "apparent" specific heat $C \equiv \partial U/\partial T|_p = T\partial S_{\mathrm{ep}}/\partial T + T_e \partial\mathcal{I}/\partial T$. Since both entropies are functions of T and $T_e(T, p)$, this can be written as $C = C_1 + C_2\,\partial T_e/\partial T$, a form postulated by Tool [12] and often used to describe the behavior in the glass formation region. Since T_e is a decreasing function of time, $\partial T_e/\partial T = \dot{T}_e/\dot{T}$ is positive in cooling, but negative for subsequent heating in the glassy state. Only when reaching the liquid state again, it becomes positive and actually exhibits an overshoot. In simple models C_1 vanishes in the glassy regime, so C is negative upon heating. In realistic glasses C is larger in cooling than in heating,

which is the same effect on top of a background C_1, that arises from uninteresting, fast equilibrium processes.

When applying these ideas to black holes, the bath is the universe filled with cosmic background radiation, presently having temperature $T_{\rm cb} \approx 2.73\,K$. The system's internal, effective temperature is the Hawking temperature. This is in agreement with the time scale argument. Black holes heavier than $10^{-18}M_\odot = 10^{15}\,g$ need more time to evaporate than the present age of the universe. For them the evaporation process, as seen by far-away observers, is so slow, that equilibration of the cosmic background radiation is a fast process.

The slow evaporation processes occur at the Hawking temperature and have as associated entropy the Bekenstein-Hawking black hole entropy S_{BH}, so eq. (8) becomes in this context

$$\text{đ}Q = T_{cb}\mathrm{d}S_m^{\rm Gs} + T_H \mathrm{d}S_{BH} \tag{9}$$

Because S_{BH} is so large, the entropy of the background radiation outside the back hole but inside the Gedanken sphere is negligible, $S_m^{\rm Gs} \ll S_{BH}$, implying $\text{đ}Q = T_H\mathrm{d}S_{BH}$. Together with eq. (6) this reproduces (4), but now it has received its non-equilibrium interpretation. Using eq. (9) and $S = S_m^{\rm Gs} + S_{BH}$ the second law (7) implies

$$(T_{\rm cb} - T_H)\mathrm{d}S_{BH} \geq 0 \tag{10}$$

Hawking radiation leads to $\mathrm{d}S_{BH} < 0$. Eq. (10) is thus satisfied as long as $T_H > T_{\rm cb}$, but not below that. One might think that $T_{\rm cb}$ plays no physical role whatsoever, and only shows up as determinator in the second law. However, the real point is that we not yet considered absorption of background radiation by the black hole. The absorption rate will be proportional to the area times the energy density, i.e., $\sim M^2T_{\rm cb}^4$. One might be tempted to find a time-dependent solution of the Einstein equations for obtaining the prefactor $\alpha_{abs}(T)$. However, what is needed is the quantum absorption process. We can solve that without doing any calculation, because it is the time-reversed evaporation process. For non-rotating, neutral holes Hawking radiation leads to a mass loss

$$\dot{M} = -\alpha_{em}\frac{\hbar c^4}{G^2M^2} \to \dot{T}_H = \frac{(8\pi)^3\alpha_{em}Gk_B^3}{\hbar^2c^5}T_H^4 \tag{11}$$

The dimensionless constant α_{em} depends on the type of particles present, and their absorption probabilities, called "oscillator strengths" in solid state systems. T_H enters through the Bose-Einstein occupation numbers (for bosons, in particular photons) or Fermi-Dirac occupation numbers (for fermions). For an uncharged, non-rotating black hole Page finds $\alpha = 5.246 \times 10^{-4}$ in the high-frequency limit, and 0.181×10^{-4} in the low frequency limit [13]. For absorption by the black hole of a photon (or a particle) from the cosmic background, the time-reversed problem shows up. It thus holds that $\alpha_{abs}(T) = \alpha_{em}(T)$, no matter the character of the particle content; for simplicity we shall now replace both by a constant. The only difference between the two situations is the temperature occurring in the occupation numbers: for Hawking emission it is T_H, while for cosmic background absorption it is $T_{\rm cb}$. The combined processes of Hawking emission and background photon absorption thus yields for a neutral, non-rotation black hole [14]

$$\dot{T}_H = \frac{(8\pi)^3\alpha Gk_B^3}{\hbar^2c^5}(T_H^4 - T_{\rm cb}^4) \tag{12}$$

It exhibits an instability at $T_H = T_{\rm cb}$, related to the fact that $C_{\rm Bek} < 0$. If there is equilibrium, and $T_{\rm cb}$ is changed a little, then T_H branches away from it.

There are two regimes. In the "classical" regime $T_H < T_{\rm cb}$ the black hole absorbs more radiation than is emits. Its entropy will increase, and $\text{đ}Q = T_H\mathrm{d}S_{BH} > 0$, but this is still in accord with the second law (10). In the "quantum" regime $T_H > T_{\rm cb}$ the black hole emits more than it absorbs. Now it holds that $\mathrm{d}S_{BH} < 0$, confirming again that heat flows from high to low temperature.

In analogy with glasses, one can define the *apparent specific heat* $C = \partial U/\partial T_{\rm cb} = \dot{U}/\dot{T}_{\rm cb}$. For black holes this object is less natural because the background temperature cannot be changed by hand. However, C does have a meaning in our expanding universe. Due to the decrease of $T_{\rm cb}$, there will be less and less background energy to be absorbed. A black hole will reach its maximal size at the moment $t = t_0$ where the temperatures match, $T_H = T_{\rm cb} = T_0$; from then on it will shrink. Around t_0 the apparent specific heat takes a form independent of $\dot{T}_{\rm cb}$, viz. $C = k_B(t-t_0)/\tau$, with characteristic time scale $\tau = \hbar/[(16\pi)^2\alpha k_BT_0]$. In the classical regime $(t < t_0)$ C is negative, while in the quantum regime it is positive.

The third law of thermodynamics concerns the entropy for $T_{\rm cb} \to 0$. We have seen already that finally all black holes evaporate, thereby lowering their configurational entropy very much, in accord with Planck's third law. What happens ultimately with the black hole has been the focus of studies by 't Hooft [15].

The entropy change of the universe is found as for black body radiation [14]

$$\frac{\mathrm{d}S_m}{\mathrm{d}S_{BH}} = \frac{T_H\mathrm{d}S_m}{\mathrm{d}U} = -\frac{T_H\mathrm{d}S_m}{\mathrm{d}U_m} = -\frac{4T_H(T_H^3 - T_{\rm cb}^3)}{3(T_H^4 - T_{\rm cb}^4)} \tag{13}$$

yielding the entropy production $\dot{S} = \dot{S}_m + \dot{S_{BH}}$

$$\dot{S} = \frac{\alpha k_B^2}{24\pi\hbar}\frac{(T_H^2 + 2T_HT_{\rm cb} + 3T_{\rm cb}^2)(T_H - T_{\rm cb})^2}{T_H^3} \tag{14}$$

Our study of models for glasses has put forward a possible universality for fluctuations that arise from mechanically coupled degrees of freedom. In terms of the four vectors $M_a = (\Omega_a, \phi)$, and $H_a = (J_a, q)$ we expect that when one writes $M_a(t,H) = M_a(T_H(T_{\rm cb},H);H)$, the following relations hold [11]

$$\chi_{ab} \equiv \frac{\partial M_a}{\partial H_b}\Big|_{T_{\rm cb}} = \chi_{ab}^{\rm fluct} + \chi_{ab}^{\rm conf} \tag{15}$$

$$\chi_{ab}^{\rm fluct} = \frac{\langle \delta M_a(t)\delta M_b(t)\rangle}{T_H(t)}; \quad \chi_{ab}^{\rm conf} = \frac{\partial M_a}{\partial T_H}\frac{\partial T_H}{\partial H_b} \tag{16}$$

The fluctuation term is the quasi-equilibrium expression that could have been guessed naively, and the configurational term is intuitively also clear. As there is no equilibrium, we do not expect that the correlation function $C_{ab}(t,t') = \langle \delta M_a(t)\delta M_b(t')\rangle$ and the response function $G_{ab}(t,t') = \partial M_a(t)/\partial H_b(t')$ depend solely on $t-t'$. Nevertheless, we do expect the validity of the quasi-equilibrium fluctuation-dissipation relation

$$\frac{\partial C_{ab}(t,t')}{\partial t'} = T_H(t')G_{ab}(t,t') \tag{17}$$

However, for the specific heat no universal quasi-equilibrium fluctuation expressions are found in glasses, and we have no reason to expect them for black holes. This is reassuring in regard of the negative "Bekenstein" specific heat. It is a challenge to test these ideas.

Let us now consider the whole universe as our system, so the entropy of the universe S_m has to be taken into account. The total entropy is $S = S_m + S_{BH}$, while eq. (8) becomes $đQ = T_{\rm cb}{\rm d}S_m + T_H{\rm d}S_{BH}$. As $đQ = 0$, the second law (7) again leads to (10), but the entropy production

$$\dot S = \frac{\alpha k_B^2}{8\pi\hbar}\,\frac{(T_H^4 - T_{\rm cb}^4)(T_H - T_{\rm cb})}{T_H^3 T_{\rm cb}} \tag{18}$$

exceeds eq. (14). The difference is due to equilibration of the emitted radiation in the universe. For small black holes, having life time less than the age of the universe, eq. (18) does not apply. They are fully evaporated before the emitted radiation can equilibrate.

In the early universe $T_{\rm cb}$ was large, and it may not have dropped from the energy balance of the black hole. Let us estimate the temperature at which the entropy of ordinary matter and the entropy of the same matter as a black hole had equal thermodynamic impact

$$T_{\rm cb}^* S_{\rm star} = T_H S_{BH} \to T_{\rm cb}^*\frac{M}{M_\odot}10^{58} = T_H\,\frac{M^2}{M_\odot^2}10^{77} \tag{19}$$

Using $T_H = T_H^\odot M_\odot/M$ we see that the masses drop from the equality, and we get $T_{\rm cb}^* = 10^{12}K$, or an energy of $75\,MeV$. This rough estimate might basically connect the entropy gain for black hole formation with disappearance of spontaneous quark-antiquark pair creation in the early universe.

In conclusion, we have shown that the non-equilibrium thermodynamics formulated for glasses also applies to black holes. It starts by considering the cosmic background radiation as heat bath, and the Hawking temperature as an internal temperature of the black hole. It is important to take into account not only the quantum evaporation of the black hole, but also its absorption of cosmic background radiation. Black holes with $T_H > 2.73\,K$ evaporate, while the ones having $T_H < 2.73\,K$ (and mass larger than $2.2\,10^{-8}M_\odot$) absorb more radiation than they emit, and continue to grow until $T_{\rm cb}$ passes through T_H.

Our approach incorporates the known properties of dynamics, and shows how the generalized second law comes into the play. Both the formation and evaporation of black holes leads to an increase of the entropy of the whole universe. Our picture involves the standard zero-entropy formulation of the third law of thermodynamics, thus putting aside the third law of black hole mechanics. To the best of our knowledge, there is no contradiction with the occurrence of negative specific heats.

Let us stress that our approach does not involve a partition sum, but merely considers known aspects of the dynamics from a thermodynamic view point. This is quite reassuring, since outside equilibrium use of the partition sum would be ill based, and it would also be ill defined.

An intriguing question is the physical meaning of the black hole entropy. If we push the analogy with solid state physics further, we may expect it to be the logarithm of the number of available states of the matter present in the black hole. Though the species-part of the entropy is much smaller than the gravitational part, we see no compelling reason why the black hole should have "forgotten" which particles it has been made of.

ACKNOWLEDGMENTS

The author is grateful for discussions with J.A.E.F. van Dongen, G. 't Hooft, L.J. van den Horn, J.M. Luck, and H. Verlinde, and for hospitality at CEA-Saclay (France), where part of this work was done.

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